



1997-98 KIRIS ASSESSMENT

Open-Response Item Scoring Worksheet

Grade 8—Mathematics Question 12

The **academic expectation** addressed by “Pizza Parlor” (Question 12) is

2.13 Students understand and appropriately use statistics and probability.

The **core content** assessed by this item includes

Probability/Statistics

Skills

- Students should be able to use counting techniques, tree diagrams, and tables to solve probability problems.

Pizza Parlor

A pizza parlor offers four toppings on its pizzas: pepperoni, sausage, mushrooms, and olives.

- a. How many different two-topping pizzas can be made? Show your work and explain your reasoning.
- b. How many different pizzas can be made with at **least** one topping? Show your work and explain your reasoning.



SCORING GUIDE

Grade 8 Mathematics

| Score | Description |
|-------|---|
| 4 | Student correctly answers part a (6 pizzas) and part b (15 pizzas) with work shown and/or explanation given. |
| 3 | Student answers both parts correctly and provides some work or explanation. OR Student answers either part a or part b incorrectly due to only one or two missing combinations. Other part is correct. |
| 2 | Student answers part a or part b correctly with work shown. OR Student answers part a correctly with little or no work shown and shows a general understanding of how to complete part b by including a few correct combinations. OR Student provides two correct answers but no work is shown. |
| 1 | Student answers part a correctly but shows no work. OR Student shows several combinations of toppings. OR Student shows minimal understanding. |
| 0 | Response is totally incorrect or irrelevant. |
| Blank | No response. |

Correct answers:

| 1 topping | 2 toppings | 3 toppings | 4 toppings |
|---|---|--|--|
| Pepperoni Sausage Mushrooms Olives | Pepperoni & Sausage Sausage & Mushrooms Mushrooms & Olives Pepperoni & Mushrooms Sausage & Olives Pepperoni & Olives | Pepperoni, Sausage & Mushrooms Pepperoni, Mushrooms, & Olives Pepperoni, Sausage, & Olives Sausage, Mushrooms, & Olives | Pepperoni, Sausage, Mushrooms & Olives |



ANNOTATED STUDENT RESPONSE

Grade 8 Mathematics

Sample 4-Point Response of Student Work

Student Response

There can be 6 different two topping pizza's made with pepperoni, sausage, mushrooms, and olives. Those are:

- | | |
|-----------------------------|---------------------------|
| 1.) pepperoni and sausage | 4.) sausage and mushrooms |
| 2.) pepperoni and mushrooms | 5.) sausage and olives |
| 3.) pepperoni and olives | 6.) olives and mushrooms |

If I make as many possible pizzas I can using at least one topping there are 15 combinations as listed below:

p = pepperoni s = sausage o = olives m = mushrooms

- | | | |
|----------|----------|---------------|
| 1. p + m | 6. m + o | 11. p,s,m + o |
| 2. p + s | 7. p | 12. p,s, + m |
| 3. p + o | 8. m | 13. p,s, + o |
| 4. s + m | 9. s | 14. p,m + o |
| 5. s + o | 10. o | 15. s, m + o |

These are all the possible combinations of pizza you can make. By jotting down different combo's I came to my conclusion.

← Student correctly answers that 6 different two-topping pizzas can be made.

← Student shows the process used to arrive at the correct number of combinations (i.e., student matches one pizza topping with other types until all combinations have been listed).

← Student correctly answers that 15 different pizzas can be made with at least one topping.

← Student shows the process used to arrive at the correct number of combinations (i.e., student assigns a variable to each topping and then uses the variables to describe and list each possible pizza).

Overall, student earns a score of 4 for correctly answering parts a and b and showing the processes used to arrive at both answers. The response demonstrates an understanding of combination problems and techniques to solve them.



ANNOTATED STUDENT RESPONSE

Grade 8 Mathematics

Sample 3-Point Response of Student Work

Student Response

a.

| | | | | |
|-----|-----|-----|---|--|
| P | S | M | O | |
| P,S | S,M | M,O | | |
| P,M | S,O | | | |
| P,O | | | | |
| | | | | |
| | | | | |

P - Pepperoni
S - Sausage
M - Mushrooms
O - Olives

This chart shows how the toppings can be paired up and how many two toppings can be put on one pizza. There are 6 pairs of two toppings.

b. I used the chart above to count how many would have at least one topping and I got 15.

Student correctly answers that 6 different two-topping pizzas can be made.

Student correctly answers that 15 different pizzas can be made with at least one topping.

Student creates an organized table to explain the process used to arrive at the correct number of combinations for part a (i.e., student matches one pizza topping with each of the other types until all combinations have been listed).

Student refers to the chart above in an attempt to explain the process used to arrive at the correct number of combinations for part b; it is not clear, however, how the student arrives at the answer.

Student earns a score of 3 for correctly answering parts a and b and showing the process used to arrive at the answer for part a.



ANNOTATED STUDENT RESPONSE

Grade 8 Mathematics

Sample 2-Point Response of Student Work

Student Response

a. My task is to find out how many 2 topping combinations can be made with 4 toppings in all.

| PEPER | MUSH |
|-------|------|
| Mush | Oliv |
| Oliv | Pep |
| PEP | Saus |
| Mush | Saus |
| Oliv | saus |

6 topping combinations in all

b. 12 pizzas because if these 6 combinations of 2 topping pizza it would double because there's only 1 topping making it 12 pizzas.

Student correctly answers that 6 different two-topping pizzas can be made.

Student incorrectly answers that 12 different pizzas can be made with at least one topping.

Student creates a table to explain the process used to arrive at the correct number of combinations for part a (i.e., student matches one pizza topping with other types until all combinations have been listed).

Student attempts to explain the process used to arrive at the number of pizzas that can be made with at least one topping. This process is incorrect because it does not account for all possible combinations (i.e., student only refers to one- and two-topping pizzas).

Student earns a score of 2 for correctly answering part a and showing the process used to arrive at the answer.



ANNOTATED STUDENT RESPONSE

Grade 8 Mathematics

Sample 1-Point Response of Student Work

Student Response

a. There can be 6, 2 topping pizzas made, for this I took four toppings divided them up and made sure all of the topping went together, this gave me my answer 6, I used my human caculator to find my exceptional answer that I agree is right.

b. There can be 4 pizzas made with at least one topping. I arrived my answer with the caculations of how many toppings I had to choose from. This is my anser from the toppings I had to chose from.

← Student correctly answers that 6 different two-topping pizzas can be made.

← Student provides a limited explanation of the process used to arrive at the correct answer (i.e., “I took four toppings divided them up and made sure all of the toppings went together”).

← Student incorrectly answers that 4 pizzas can be made with at least one topping and attempts to explain how this incorrect conclusion is reached.

Student earns a score of 1 for answering part a correctly.



INSTRUCTIONAL STRATEGIES

Grade 8 Mathematics

The open-response item “Pizza Parlor” is designed to assess students’ understanding of combination problems and the techniques used to solve them. The instructional strategies below present ideas for helping students explore and master these concepts.

Using both concrete (e.g., colored tiles) and representational models (e.g., tree diagram, t-table, organized list), show students how to systematically determine combinations. (Note: Include combinations with different parameters. For example, a combination problem similar to “Pizza Parlor” could use such parameters as “must include pepperoni,” or must use three of four toppings.)

Model strategies for effectively communicating mathematical reasoning both verbally and in writing.

Teach students a variety of strategies for organizing information (e.g., tables, charts, graphs, highlighting, underlining) to help them solve combination problems. Point out to students that these strategies can be used to help them write equations, check their reasoning and the reasonableness of their answers, document their thinking, and explain their work to others.

Provide opportunities for students to work individually, in pairs, and/or in groups to complete (with teacher support and guidance) any or all of the following activities:

- Use two different sets of concrete models to practice matching objects to determine the number of combinations that can be made.
- Practice organizing combinations in a variety of ways. Include tree diagrams, organized lists, and tables.
- Practice finding combinations with a variety of imposed or extended parameters (e.g., after finding the number of shirt/tie combinations that can be made from three ties and three shirts, find the number of combinations that can be made if each must contain a blue tie).
- Discuss and write about different techniques for finding combinations. (This activity can help students develop and/or refine their ability to effectively communicate their mathematical thinking both verbally and in writing. Prior to these activities, the teacher can model strategies for communicating mathematical thinking.)